1 Terminology and Definition

class	terminology	definition	properties*
field property	conservative	$\oint_C \mathbf{E}\cdot d\ell = 0$ $\nabla\times\mathbf{E} = 0$ $\mathbf{E} = -\nabla V$	=irrotional/potential field
	solenoidal	$\oint_S \mathbf{E}\cdot \mathbf{ds} = 0$ $\nabla\cdot\mathbf{E} = 0$ $\nabla^2 V = 0$	=source-less/tubular
	uniquely defined	both $\nabla\times$ and $\nabla\cdot$ defined	
electric field (conservative)	electrostatic field	$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\mathbf{E} + \mathbf{P}$	$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$
	steady current field	$\mathbf{J} = \sigma\mathbf{E}$	$\nabla^2 V = 0$
	induced field	$\mathbf{E}_{\text{ind}} = -\frac{\partial\mathbf{A}}{\partial t}$	non-conservative, tubular
magnetic field (source-less)	magnetostatic field	$\mathbf{H} = \frac{\mathbf{B}}{\mu} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$	$\nabla^2\mathbf{A} = -\mu_0\mathbf{J}, \quad \nabla^2 U_m = 0$
time-varying electric&magnetic field		$\mathbf{E} = -\nabla V - \frac{\partial\mathbf{A}}{\partial t}$	interdependent
plane waves	plane wave	$\hat{E}\times\hat{H} = \hat{k}$	far from radiating source
	uniform plane wave	$E(x), H(x)$	uniform borderless media
	TEM		orthogonal
guided waves	TE mode	$E_{0z} = 0$	$TE_{10}, TE_{20}, TE_{01}, TE_{11}\dots$
	TM mode	$H_{0z} = 0$	$TM_{11}, TM_{21}, \dots$
	degenerated modes	TE_{mn} and TM_{mn}	
	TEM	$k_c^2 = \gamma^2 + k^2 = 0$	$\mathbf{E}, \mathbf{H}\perp\hat{k}$
network	reciprocal	$[\mu], [\varepsilon], [Z], [Y], [S]$ symmetric	
	lossless	$[Z], [Y]$ purely imaginary, $[S]^T[S]^* = [U]$ no R or G, reflection = incidence	
	port	$V_i = V_i^+ + V_i^-$	incidence+reflection

2 Time-Invariant Fields

2.1 Electrostatics

formulas	form	expression
Coulomb's law		$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$
electric field	def	$\vec{\mathbf{E}} = \sum \frac{1}{4\pi\varepsilon}\frac{q_i}{r^2}\hat{r}$
electric potential	def	$\mathbf{E} = -\nabla V$
	cal.	$\Delta V_{ab} = -\int_b^a \mathbf{E}\cdot \mathrm{d}l$
	point	$V_a = \frac{q}{4\pi\varepsilon r}$

2.2 Steady electric current

formulas	form	expression
current density	def	$\mathbf{J} = \rho\mathbf{v} = \sigma\mathbf{E}$
current	def	$I = \int_S \mathbf{J}\cdot\mathbf{ds} = \int \mathbf{J_s}\cdot \mathrm{d}\ell$
Equation of Continuity	diff	$\nabla\cdot\mathbf{J} = -\frac{\partial\rho_v}{\partial t} = 0$
	int	$\oint_S \mathbf{J}\cdot\mathbf{ds} = \int_V -\frac{\partial\rho_v}{\partial t}\mathrm{d}v = 0$
	homo medium	$\nabla\cdot\mathbf{E} = 0 \Leftrightarrow \nabla^2 V = 0$
Ohm's law	diff	$\mathbf{J} = \sigma\mathbf{E}$
	low freq.	$V = RI$
Joule's law	diff	$P = \mathbf{J}\cdot\mathbf{E}$
	int	$P = \int_V \mathbf{J}\cdot\mathbf{E}\mathrm{d}v$
	low freq.	$P = IV$

2.3 Magnetostatics

formulas	form	expression
force		$\mathrm{d}\mathbf{F} = I\mathrm{d}\ell \times \mathbf{B}$
magnetic dipole	approx	
magnetic moment	def	$\mathbf{m} = IA\hat{n} = NI\mathbf{A}$
magnetic torque	def	$\mathbf{T} = \mathbf{m} \times \mathbf{B}$
Biot-Savart law		$\mathrm{d}\mathbf{B} = \frac{\mu I}{4\pi r^2}\mathrm{d}\ell \times \hat{r} = \frac{\mu \mathbf{J} \times \hat{r}}{4\pi r^2}\mathrm{d}v$
vector potential	eq.	$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$
	cal.	$\mathbf{A} = \frac{\mu}{4\pi}\int_C \frac{\mathbf{J}\mathrm{d}\ell}{r} = \frac{\mu}{4\pi}\int_V \frac{\mathbf{J}\mathrm{d}v}{r}$
	cal.**B**	$\mathbf{B} = \nabla \times \mathbf{A}$
		$\Phi_B = \oint_C \mathbf{A} \cdot \mathrm{d}\ell$
scalar potential (current-less)	def	$\mathbf{H} = -\nabla U_m$
	mmf	$U_m(P) = \int_P^\infty \mathbf{H} \cdot \mathrm{d}\ell$
	eq.	$\nabla^2 U_m = 0$

2.4 Charge/current distributions and their fields

wire	$E = \frac{\lambda}{2\pi\varepsilon r}$	$B = \frac{\mu I}{2\pi r}$
infinite plane	$E = \frac{\sigma}{2\varepsilon}$	$B = \frac{\mu i}{2}$
long solenoid		$B_{in} = \mu n I$
toroid		$B_{in} = \frac{\mu N i}{2\pi r}$
ring	$E = \frac{Qx}{4\pi\varepsilon(x^2+R^2)^{3/2}}$	$B = \frac{\mu I R^2}{2(R^2+x^2)^{3/2}}$
disk	$E = \frac{\sigma}{2\varepsilon}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$	

3 Time-Varying Fields

using FT, we have

$$\begin{aligned}\mathbf{E}(r,t) &= \mathrm{Re}\{[\tilde{E}_x(r)\vec{\mathbf{a}}_x + \tilde{E}_y(r)\vec{\mathbf{a}}_y + \tilde{E}_z(r)\vec{\mathbf{a}}_z]e^{\mathrm{j}\omega t}\} \\ &= \mathrm{Re}[\tilde{\mathbf{E}}(r)e^{\mathrm{j}\omega t}]\end{aligned}$$

where $\tilde{\mathbf{E}}(r) = \tilde{E}_x(r)\vec{\mathbf{a}}_x + \tilde{E}_y(r)\vec{\mathbf{a}}_y + \tilde{E}_z(r)\vec{\mathbf{a}}_z$ Complex amplitude and complex vector depend on position r and are time-dependent; Transient field vector and components are real functions.

3.1 Summary: Maxwell's equations

electric Gauss's law	diff	$\nabla \cdot \mathbf{D} = \rho_v$
	int	$\oint_S D \cdot \mathrm{d}\mathbf{s} = \int_V \rho_v \mathrm{d}v$
magnetic Gauss's law	diff	$\nabla \cdot \mathbf{B} = 0$
	int	$\oint_S \mathbf{B} \cdot \mathrm{d}\mathbf{s} = 0$
Ampere's loop law	diff	$\nabla \times \mathbf{H} = \mathbf{J}_v + \frac{\partial \mathbf{D}}{\partial t}$
	int	$\oint_C \mathbf{H} \cdot \mathrm{d}\ell = \int_S (\mathbf{J}_v + \frac{\partial \mathbf{D}}{\partial t}) \cdot \mathrm{d}\mathbf{s}$
	FT	$\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}}_v + \mathrm{j}\omega\tilde{\mathbf{D}}$
Faraday's law (induced + motional emf)	diff	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B})$
	int	$\mathscr{E} = \oint_C \mathbf{E} \cdot \mathrm{d}\ell = -\frac{\mathrm{d}}{\mathrm{d}t}\int_S \mathbf{B} \cdot \mathrm{d}\mathbf{s}$ $= -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot \mathrm{d}\mathbf{s} + \oint_C (\mathbf{v} \times \mathbf{B}) \cdot \mathrm{d}\ell$
	FT	$\nabla \times \tilde{\mathbf{E}} = -\mathrm{j}\omega\tilde{\mathbf{B}}$
Lorentz force		$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

3.2 Constitutive relationships

Constitutive equations (linear, homogeneous, and isotropic medium) are given in section1. How to calculate (equivalent) bound charge/current?

polarized dielectric	$V(P) = \frac{1}{4\pi\varepsilon_0}\left(\oint_S \frac{\rho_{sb}}{r}\mathrm{d}s + \int_V \frac{\rho_{vb}}{r}\mathrm{d}v\right)$	S, Q	$\rho_{sb} = \mathbf{P} \cdot \hat{n}$
		V, Q	$\rho_{vb} = -\nabla \cdot \mathbf{P}$
magnetized material	$\mathbf{A} = \frac{\mu_0}{4\pi}(\int_V \frac{\mathbf{J}_{mv}}{R}\mathrm{d}v + \int_S \frac{\mathbf{J}_{ms}}{R}\mathrm{d}s)$	S, I	$\mathbf{J}_{ms} = \mathbf{M} \times \hat{n}$
		V, I	$\mathbf{J}_m = \nabla \times \mathbf{M}$
eq. magnetic charge	$U_m = \frac{1}{4\pi}\left(\oint_S \frac{\rho_{ms}}{r}\mathrm{d}s + \int_V \frac{\rho_{mv}}{r}\mathrm{d}v\right)$	S, Q	$\rho_{ms} = \mathbf{M} \cdot \hat{n}$
		V, Q	$\rho_m = -\nabla \cdot \mathbf{M}$

3.3 Boundary conditions

ANALOGY **D**&**J**			
displacements	$\mathbf{D}$	$\mathbf{J}$	free charge
related to	electrostatic	steady current	
definition	$\mathbf{D} = \varepsilon \mathbf{E}$	$\mathbf{J} = \sigma \mathbf{E}$	
boundary conditions	$D_{n1} = D_{n2}$	$J_{n1} = J_{n2}$	$\rho_s = D_{n1} - D_{n2}$ $= J_n(\frac{\varepsilon_1}{\sigma_1} - \frac{\varepsilon_2}{\sigma_2})$
	$E_{t1} = E_{t2}$		
MAGNETIC FIELD&POTENTIAL			
magnetic	$\mathbf{H}$	$\mathbf{U_m}, \mathbf{A}$	free current
boundary conditions	$H_{t1} = H_{t2}$	continuous	$\mathbf{J}_s = H_{t1} - H_{t2}$ $= \hat{n} \times (\mathbf{H}_1 - \mathbf{H}_2)$
	$B_{n1} = B_{n2}$		
ELECTRIC-MAGNETIC COUNTERPARTS			
flux density	$\mathbf{D}$	flux intensity	$\mathbf{B}$
field intensity	$\mathbf{E}$	field intensity	$\mathbf{H}$
polarization	$\mathbf{P}$	magnetization	$\mathbf{M} = \frac{\sum \mathbf{m}}{\Delta v}$

When is there free charge/current at the boundary?

Perfect Conductor $(\sigma = \infty, \varepsilon = 0)$ inside, electromagnetic fields $\mathbf{E} = \mathbf{B} = 0$; on its surface, both ρ_s and J_s can exist, $\mathbf{E} \parallel \hat{n}$ and $\mathbf{H} \perp \hat{n}$.

A Conductor $(\sigma < \infty, \varepsilon = 0)$ inside, time-varying fields can exist; hence $J_s = 0$, but ρ_s can exist between a conductor and a perfect dielectric.

Two Perfect Dielectric $(\sigma = 0, \varepsilon)$ $J_s = 0$. However, $\rho_s = 0$ unless the charge is physically placed at the interface.

Ferromagnetic $(\mu = \infty)$

3.4 Energy&Work

energy densities	$\mathbf{E}$	$W = q\Delta V = \int_V \frac{1}{2}\rho_v V \mathrm{d}v$ $w_e = \frac{1}{2}\mathbf{D}\cdot\mathbf{E} = \frac{1}{2}\varepsilon E^2$
	$\mathbf{B}$	$W = \sum \frac{1}{2} I_i \Psi_i = \frac{1}{2}\int_V A \cdot \mathbf{J} \mathrm{d}v$ $w_m = \frac{1}{2}\mathbf{B}\cdot\mathbf{H} = \frac{1}{2}\mu H^2$
Poynting's theorem (energy conservation)	diff	$\nabla \cdot (\mathbf{E} \times \mathbf{H}) + \mathbf{J}\cdot\mathbf{E} + \mathbf{H}\cdot\frac{\partial \mathbf{B}}{\partial t} + \mathbf{E}\cdot\frac{\partial \mathbf{D}}{\partial t} = 0$
		in homogeneous, isotropic medium ⇓
	int	$\oint_S \mathbf{S}\cdot \mathrm{d}\mathbf{s} + \int_V \mathbf{J}\cdot\mathbf{E}\ \mathrm{d}v + \frac{\partial}{\partial t}\int_V (w_m + w_e)\ \mathrm{d}v = 0$
	FT	$\langle \mathbf{S} \rangle = \int_T \mathbf{S} \mathrm{d}t = \Re\left\{\frac{1}{2}\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\right\}$ $w_m = \frac{\mu}{4}\lvert\tilde{\mathbf{H}}\rvert^2 = \frac{LI^2}{4}, w_e = \frac{\varepsilon}{4}\lvert\tilde{\mathbf{E}}\rvert^2 = \frac{CV^2}{4}$ $\nabla \cdot \tilde{\mathbf{S}} + \frac{1}{2}\tilde{\mathbf{E}}\cdot\tilde{\mathbf{J}}^* + \mathrm{j}\omega(\frac{1}{2}\tilde{\mathbf{B}}\cdot\tilde{\mathbf{H}}^* - \frac{1}{2}\tilde{\mathbf{E}}\cdot\tilde{\mathbf{D}}^*) = 0$

More about Poynting's theorem:

1. **Poynting vector** $\mathbf{S} = \mathbf{E} \times \mathbf{H}$, represents *the instantaneous EM power crossing the closed surface S.* If this integral is positive, the net power is flowing out of the volume; flowing in if negative.

2. $\mathbf{E} \cdot \mathbf{J}$ represents *the power supplied to the charged particles by the electric field.* When positive, the field is doing work. In a conductor, $\mathbf{J} = \sigma\mathbf{E}$, this term represents power dissipation or ohmic power loss (Joule loss, mostly heat).

3. $-\frac{\partial w}{\partial t}$ represents *the change rate of stored magnetic/electric energy.* For static field this term equals 0.

4 Solving for potential

for static fields,

Poisson eq	$\nabla^2 V = -\frac{\varrho}{\varepsilon}$	$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$
Laplace eq	$\nabla^2 V = 0$	$\nabla^2 U_m = 0$
cartesian	$U(x,y,z) = \sum_{n=1}^{N} X_n(x) Y_n(y) Z_n(z)$	
cylindrical	$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \phi}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$ $U = R(\rho)\,\Phi(\phi) = (A_0 + B_0 \ln \rho) + \sum_{n=1}^{\infty}(A_n \rho^n + B_n \rho^{-n})(C_n \sin n\phi + D_n \cos n\phi)$	
spherical	$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial U}{\partial r}\right) + \frac{1}{r^2 \sin\theta} \cdot \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial U}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta} \cdot \frac{\partial^2 U}{\partial \varphi^2} = 0$ $U(r,\theta) = \sum_{n=0}^{\infty}(A_n r^n + B_n r^{-(n+1)}) P_n(\cos\theta)$, where $\lvert P_n(\cos\theta)\rvert^2 = \frac{2}{2n+1}$	

for time-varying fields,

	transient	complex
Dynamic Potential	$\mathbf{B} = \nabla \times \mathbf{A}$ $\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$	$\mathbf{B} = \nabla \times \mathbf{A}$ $\mathbf{E} = -\frac{\nabla(\nabla \cdot \mathbf{A})}{\mathrm{j}\omega\mu\varepsilon} - \mathrm{j}\omega \mathbf{A}$
Lorentz gauge	$\nabla \cdot \mathbf{A} = -\mu\varepsilon \frac{\partial V}{\partial t}$	$\nabla \cdot \mathbf{A} = -\mathrm{j}\omega\mu\varepsilon V$
Darren Bell eq	$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\varrho}{\varepsilon}$ $\nabla^2 \mathbf{A} - \mu\varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}$	$\nabla^2 V + \omega^2 \mu\varepsilon V = -\frac{\varrho}{\varepsilon}$ $\nabla^2 \mathbf{A} + \omega^2 \mu\varepsilon \mathbf{A} = -\mu \mathbf{J}$
Helmholtz Equation (source-less region)	$\nabla^2 \bar{E} = \mu\sigma \frac{\partial \mathbf{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$ $\nabla^2 \mathbf{H} = \mu\sigma \frac{\partial \mathbf{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \mathbf{H}}{\partial t^2}$	$\nabla^2 \vec{E} + k^2 \vec{E} = 0$ $\nabla^2 \vec{H} + k^2 \vec{H} = 0$ $k^2 = \omega\mu(\omega\varepsilon - \mathrm{j}\sigma)$

5 EM Waves

5.1 Plane waves

when $\sigma = 0$ (lossless) we have $k = \beta = \sqrt{\omega^2 \mu\varepsilon}$

wave equation	$\frac{\mathrm{d}^2 E_x}{\mathrm{d}z^2} = \mu\varepsilon \frac{\partial^2 \bar{E}_x}{\partial t^2}$ $\frac{\mathrm{d}^2 H_y}{\mathrm{d}z^2} = \mu\varepsilon \frac{\partial^2 H_y}{\partial t^2}$	$\mathbf{E} = E_0 e^{-\mathrm{j}kz}$ $\tilde{E}_x(z) = E_{xf} e^{-j(kz - \theta_{xf})} + E_{xb} e^{j(kz + \theta_{xb})}$
	$E_x(z,t) = E_{xf}\cos(\omega t - kz + \theta_{xf}) + E_{xb}\cos(\omega t + kz + \theta_{xb})$	
orthogonality	$\hat{E} \times \hat{H} = \hat{k}, \mathbf{H} = \frac{\mathbf{k} \times \mathbf{E}}{\omega\mu}, \mathbf{E} = \frac{\mathbf{H} \times \mathbf{k}}{\omega\varepsilon - \mathrm{j}\sigma}$	
forward&backward	$E_x(z,t) = E_x^+(z,t) + E_x^-(z,t)$ $H_y(z,t) = H_y^+(z,t) + H_y^-(z,t)$ $\frac{E_x^+}{H_y^+} = \sqrt{\mu/\varepsilon} \approx \eta, \frac{E_x^-}{H_y^-} = -\sqrt{\mu/\varepsilon} \approx -\eta$	
velocity	$\mathbf{k} = \frac{2\pi}{\lambda}\hat{k} = \sqrt{\omega^2 \mu\varepsilon}\hat{k}, v = \frac{1}{\sqrt{\mu\varepsilon}}$	
energy	$w_e = \frac{1}{2}\varepsilon(E_x^+)^2 = \frac{1}{2}\mu(H_y^+)^2 = w_m$ $S^+(z,t) = vw\hat{k} = v(w_e + w_m)\hat{z}, \langle \mathbf{S} \rangle = \frac{\lvert \mathbf{E} \rvert^2}{2\omega\mu}\hat{k} = \frac{E^2}{2\eta}\hat{k}$	

when $\sigma \neq 0$ (with loss), $k = \beta - \mathrm{j}\alpha = \sqrt{\omega^2 \mu\varepsilon - \mathrm{j}\omega\mu\sigma}$, intrinsic/ wave impedance $\eta = \frac{\omega\mu}{k} = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{1 - \mathrm{j}\frac{\sigma}{\omega\varepsilon}}}$

5.2 Polarization

linearly polarized E_x, E_y in phase ($\theta_x = \theta_y$). $\mathbf{E}$ is sinusoidal

elliptically polarized right-handed, E_x leads; left-handed, E_x lags. $\mathbf{E}$ is spiral

circularly polarized right-handed, E_x leads by $\pi/2$; left-handed, E_x lags by $\pi/2$. $\mathbf{E}$ is spiral

5.3 Dispersion

To $\mathbf{E}, \mathbf{H}$ at high freq., media parameters change, $\mathbf{P}, \mathbf{M}$ and charge movement lag. The loss of a medium is measured by **loss tangent**

$$\tan\delta_e = \frac{\varepsilon''}{\varepsilon'}, \varepsilon = \varepsilon' - \mathrm{j}\varepsilon''$$

$$\tan\delta_m = \frac{\mu''}{\mu'}, \mu = \mu' - \mathrm{j}\mu''$$

5.4 Velocities

In dispersive media (where the phase velocity varies with freq.), the group velocity may differ from the phase velocity.

phase velocity	group velocity
of a point of const. phase on the wave; of carrier	of the envelope of the wave; of energy propagation; of modulation
$v_p = \frac{\omega}{\beta}$	$v_g = \frac{\mathrm{d}\omega}{\mathrm{d}\beta} \leq v_p$
$v_p v_g = \frac{1}{\mu\varepsilon} = c^2 // v_p = v_g$ when no dispersion (e.g. in dielectric)	

5.5 Guided waves

For ideal conductors, power is transmitted via fields and not through the conductors themselves; for loss conditions, power in the conductors is lost as heat. classification: TEM-type lines (coaxial cable, microstrip line, stripline), non-TEM lines (waveguides)

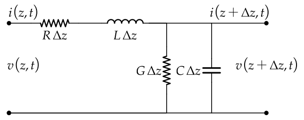
rectangular waveguide single conductor -> no TEM mode The mode is cutoff when $k = k_c^{mn}$

$k_c^{mn} = \sqrt{(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2}$	$E_z = 0$
$f_c^{mn} = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}$	$H_z = A\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$
$\beta, k_z = \sqrt{k^2 - k_c^2}$	$E_x = \frac{j\omega\mu n\pi}{k_c^2 b}A\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$
$\lambda_g = \frac{2\pi}{\beta}, v_p = \frac{\omega}{\beta}$	$E_y = \frac{-j\omega\mu m\pi}{k_c^2 a}A\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$
$Z_{TE} = \frac{k\eta}{\beta}$	$H_x = \frac{j\beta m\pi}{k_c^2 a}A\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$
$Z_{TM} = \frac{\beta\eta}{k}$	$H_y = \frac{j\beta n\pi}{k_c^2 b}A\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$

cavity resonator When a mode (standing wave) can exist at the resonant frequency, it traps energy at that frequency.

$$f_{mnp} = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2 + (\frac{p}{d})^2}$$

6 Transmission Line Theory and Network Analysis



Telegrapher's Equations

$$\text{trans.}\begin{cases}\frac{\partial v}{\partial z}=-Ri-L\frac{\partial i}{\partial t}\\ \frac{\partial i}{\partial z}=-Gv-C\frac{\partial v}{\partial t}\end{cases}\quad \text{freq.}\begin{cases}\frac{dV}{dz}=-(R+j\omega L)I\\ \frac{dI}{dz}=-(G+j\omega C)V\end{cases}$$

⇩ decouple

$$\frac{\partial^2 v}{\partial z^2}-(RG)v-(RC+LG)\frac{\partial v}{\partial t}-LC\left(\frac{\partial^2 v}{\partial t^2}\right)=0$$

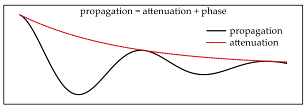
$$\frac{d^2V}{dz^2}=(R+j\omega L)(G+j\omega C)V=(ZY)V=\gamma^2V$$

⇩ solve

$$V(z)=V_0^+e^{-\gamma z}+V_0^-e^{+\gamma z}=V_0^+e^{-\alpha z}e^{-j\beta z}+V_0^-e^{+\alpha z}e^{+j\beta z}$$

$$v^+(z,t)=\mathrm{Re}\{V(z)e^{j\omega t}\}=|V_0^+|e^{-\alpha z}\cos(\omega t-\beta z+\phi)$$

即随距离指数衰减的振荡波



lossless case $(R=G=0)$

propagation const. $\gamma=\sqrt{(R+j\omega L)(G+j\omega C)}=\alpha+j\beta$

phase const. $\beta=\frac{2\pi}{\lambda_g}=\omega\sqrt{LC}$ guided wavelength λ_g

phase velocity $v_p=\frac{\omega}{\beta}=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{\mu\varepsilon}}$

characteristic impedence $\frac{V_0^+}{I_0^+}=-\frac{V_0^-}{I_0^-}=Z_0=\sqrt{\frac{R+j\omega L}{G+j\omega C}}=\sqrt{\frac{L}{C}}$

reflection coefficient $\Gamma(z)=\Gamma(-l)=\frac{V^-(z)}{V^+(z)}=\Gamma(0)e^{-2\gamma l}$

指标

return loss $\mathrm{RL}=-20\log|\Gamma_L|\ \mathrm{dB}\in[0,\infty]$

voltage standing wave ratio $\mathrm{VSWR}=\frac{V^+_{\max}}{V^+_{\min}}=\frac{1+|\Gamma_L|}{1-|\Gamma_L|}\in[1,\infty]$

transmission coefficient $T=1+\Gamma=1+\frac{Z_1-Z_0}{Z_1+Z_0}$

insertion loss $\mathrm{IL}=-20\log|T|\mathrm{dB}$

$1\ \mathrm{Np}=10\log e^2=8.68589\mathrm{dB}$

Quarter-Wave Transformer

$l=\frac{\lambda_g}{4}+\frac{n\lambda_g}{2},\ Z_{0T}=\sqrt{Z_0Z_L}$

transform Z_L in an inverse manner

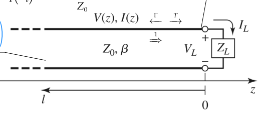
$\Gamma_l=\Gamma(-l)=\Gamma(0)e^{-2\gamma l}$

$Z_{in}=Z(-l)=Z_0\left(\frac{1+\Gamma_Le^{-2\gamma l}}{1-\Gamma_Le^{-2\gamma l}}\right)$

$=Z_0\left(\frac{Z_L+Z_0\tanh(\gamma l)}{Z_0+Z_L\tanh(\gamma l)}\right)=Z_0\left(\frac{Z_L+jZ_0\tan(\beta l)}{Z_0+jZ_L\tan(\beta l)}\right)$

$V(z)=V(-l)=V_0^+e^{\gamma l}+V_0^-e^{-\gamma l}$

$I(-l)=\frac{V_0^+e^{\gamma l}-V_0^-e^{-\gamma l}}{Z_0}$



$\Gamma_L=\Gamma(0)=\frac{V_0^-}{V_0^+}=\frac{Z_L-Z_0}{Z_L+Z_0}$

load $\begin{cases}\text{match } Z_L=Z_0,\ \Gamma(0)=0,\ \text{no reflected wave}\\ \text{short } Z_L=0,\ Z_{in}=jZ_0\tan\beta l\\ \text{open } Z_L=\infty,\ Z_{in}=-jZ_0\cot\beta l\end{cases}$

line $\begin{cases}l=\lambda_g/2,\ Z_{in}=Z_L\\ l=\lambda_g/4,\ Z_{in}=\frac{Z_0^2}{Z_L}\end{cases}$

$$P(z)=\frac{1}{2}\mathrm{Re}\{VI^*\}=\frac{1}{2}\mathrm{Re}\left\{\frac{|V_O^+|^2}{Z_0^*}(1+\Gamma_Le^{2j\beta z})(1-(\Gamma_Le^{2j\beta z})^*)\right\}$$

$$\approx\frac{1}{2}\frac{|V_O^+|^2}{Z_0}e^{2\alpha l}(1-|\Gamma_L|^2e^{-4\alpha l})=\frac{1}{2}\frac{|V_O^+|^2}{Z_0}(1-|\Gamma_L|^2)$$

intrinsic impedance $\eta=\sqrt{\mu/\varepsilon}$ of medium material, equals Z_ω for plane waves

wave impedance $Z_\omega=E_t/H_t$, e.g. $Z_{\mathrm{TEM}},Z_{\mathrm{TM}},Z_{\mathrm{TE}}$

characteristic impedance $Z_0=V^+/I^+$, unique for TEM wave, varyingly for TE and TM waves

short circuit: $V_i^++V_i^-=0$; open circuit: $I_i^++I_i^-=0$
Z_{ii} input impedance seen looking into port i
$S_{ii}=\Gamma_i,S_{ij}=T_{ij}$ when matched

$$P^+=\frac{1}{2}V^+I^{+*},P_{avg}=\frac{1}{2}\Re\{[V]^T[I]^*\}$$

6.1 Scattering Matrix: Incident and Reflected Voltage

At high freq. measurements involve the magnitude and phase of a wave traveling in a given direction or of a standing wave.

$$[V^-]=[S][V^+],[S]=([Z]+[U])^{-1}([Z]-[U])$$

$S_{ij}=\frac{V_i^-}{V_j^+}|_{\Gamma=0}$: driving port j with an incident V_j^+, all other ports matched to avoid reflections $(V^+=0)$, measure the reflected wave amplitude V_i^- at port i

7 Communication Systems

7.1 Antennas

A transmitting antenna is a device that converts a guided electromagnetic wave on a transmission line into a plane wave propagating in free space. Antennas are bi-directional and can be used for both transmit and receive functions. Near field: reactive, depends on r; far field: radiating, independent of r.

bandwidth	BW	$VSWR < 1.5$
far-field distance		$R_{ff} = \frac{2D^2}{\lambda}$
radiation pattern (far field, normalised)	field	$F(\theta, \phi) = \frac{\lvert E(\theta,\phi)\rvert}{E_{\max}}$
	power	$P(\theta, \phi) = \frac{S(\theta,\phi)}{S_{\max}} = \lvert F(\theta, \phi)\rvert^2$
$\mathbf{E} = (\hat{\theta} F_\theta + \hat{\phi} F_\phi) e^{-jk_0 r}/r, \quad \eta_0 = E_\theta / H_\phi = -E_\phi / H_\theta$ $P_{rad} = \oint_S U(\theta, \phi) \mathrm{d}s, \quad P_r = G_r A_{eff} S_{\text{avg}}, \quad S_{\text{avg}} = \frac{G_t P_t}{4\pi r^2}$		
directivity	$\frac{\text{main beam}}{\text{average}}$	$D(\theta, \phi) = \frac{U_{\max}}{U_{\text{avg}}} = \frac{4\pi U_{\max}}{P_{rad}} \leq \frac{4\pi A}{\lambda^2}$
radiation efficiency	resistive loss	$\eta_{rad} = \frac{P_{rad}}{P_{in}} = 1 - \frac{P_{loss}}{P_{in}}$
gain	$\frac{\text{directional}}{\text{isotropic rad.}}$	$G = \eta_{rad} D$
effective aperture area	$\frac{P_L}{P_{in}}$	$A_{eff}(\max) = \frac{\lambda^2}{4\pi} D$
antenna noise temp.	noise delivered	$T_A = \eta_{rad} T_B + (1 - \eta_{rad}) T_P$
(SNR)		$G/T(\text{dB}) = 10 \log{(G/T_A)} \text{dB/K}$

7.2 Noise

thermal noise(thermal vibration of bound charges), shot noise (random fluctuations of charge carriers), flicker noise($1/f$ noise, varies inversely with frequency), plasma noise, quantum noise

$$P_n = \frac{V_n^2}{4R} = kT(BW), \quad T_{eq} = \frac{N_o}{Gk(BW)}, \quad NF = \frac{S_i/N_i}{S_o/N_o} = 1 + \frac{T_e}{290\text{K}}$$

$$T = T_{eq1} + \frac{T_{eq2}}{G_1} + \frac{T_{eq3}}{G_1 G_2} + \cdots, \quad NF = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3}{G_1 G_2} + \cdots$$

when matched $G_{21} = |S_{21}|^2$, $NF = 1 + \frac{1 - G_{21}}{G_{21}} \frac{T}{T_0}$

7.3 Link Budget

transmitted antenna line loss	$L_t = \alpha$
path loss	$L_0(\text{dB}) = 20 \log{(4\pi r/\lambda)} > 0$
atmosphere attenuation	L_A
receive antenna line loss	L_r
receive power	$P_r(\text{dBm}) = (P_t + G_t + G_r) - (L_t + L_0 + L_r)$
impedance mismatch loss	$L_{\text{imp}}(\text{dB}) = -10 \log(1 - \lvert\Gamma\rvert^2) \geq 0$
link margin	$\text{LM} = P_r - P_r(\text{mim})$

The Friis Formula For long distance comm, wireless radio links better than wired links TL.

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi r}\right)^2, \quad \text{Effective Isotropic Radiated Power} = P_t G_t$$

For given freq., range, G_r, the received power is proportional to EIRP of the transmitter, which can only be increased by increasing P_t or G_t.

$$T_{TL+r} = (L_t - 1) T_P + L_t T_r,$$

$$\frac{S_o}{N_o} = \frac{S_i}{k(BW)(\eta_{rad} T_B + (1 - \eta_{rad}) T_P + (L_t - 1) T_P + L_t T_r)}$$